MATH 3060 Tutorial 8

Chan Ki Fung

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1 Questions of this tutorial

1. Let $f : X \to Y$ be a continuous map between metric spaces. Suppose f has the property that $f^{-1}(K)$ is compact for any compact $K \subset Y$, show that $f(F)$ is closed for any closed $F \subset X$.

2. Let $U \subset \mathbb{R}^n$, $V \subset \mathbb{R}^m$ be open subsets. Let $f : U \to V$ be a continuous map. Suppose f has the property that $f^{-1}(p)$ is compact for any $p \in V$, and the image $f(F)$ is closed for any closed $F \subset U$. Show that $f^{-1}(K)$ is compact for any compact $F \subset V$.

3. Let G be a closed and bounded subset of $\mathbb{R}^n$, and $(f_n) \in C(G)$ is a sequence of function. Suppose $f_n \to f$ pointwise, show that $f_n \to f$ uniformly.

4. Show that the boundedness assumption of the Ascoli's theorem can be weakened to pointwise boundedness.

5. (a) Let $g : [0, 1] \to \mathbb{R}$ be a (monotonically) increasing function, show that the set of discontinuity of g is countable.

   (b) Let $f_n : [0, 1] \to \mathbb{R}$ be a sequence of increasing functions, show that there is a (pointwise) converging subsequence.